# Kinosaki Algebraic Geometry Symposium 2019 $({\rm Revised})$

Date	:	2019  October  21  (Mon) - 2019  October  25  (Fri)
Venue	:	Kinosaki International Arts Center
	:	(Yushima 1062, Kinosaki, Toyo-oka, Hyogo 669-6101)
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# Program

# Oct. 22 (Tue)

$9:30 \sim 10:30$	Hideo Kojima (Niigata University) Some results on open algebraic surfaces of logarithmic Kodaira dimension zero
$10:45 \sim 11:45$	Masaru Nagaoka (University of Tokyo) Compactifications of affine homology 3-cells into quadric fibrations
$13:30 \sim 14:30$	Shihoko Ishii (Tsinghua University/University of Tokyo) A description of divisors by arcs and its applications
$14:45 \sim 15:45$	Joonyeong Won (Korea Institute for Advanced Study) Sasaki-Einstein metrics on simply connected rational homology 5-spheres
$16{:}00{\sim}17{:}00$	Poster session
$18{:}00{\sim}20{:}00$	Banquet
Oct. 23 (Wed)	

$9:15 \sim 10:15$	Takuzo Okada (Saga University) Birationally superrigid Fano threefolds of codimension 4
$10:30 \sim 11:30$	Ken-ichi Yoshikawa (Kyoto University) Enriques 2 <i>n</i> -folds and analytic torsion

12:00  $\sim 17{:}00$   $\,$  Free discussion

## Oct. 24 (Thu)

$9:15 \sim 10:15$	Gregory Kumar Sankaran (University of Bath) Locally symmetric varieties and holomorphic symplectic manifolds
$10:30 \sim 11:30$	Shigefumi Mori (Kyoto University Institute for Advanced Study) Extremal contractions of 3-folds with fiber dimension 1
13:30 ~ 14:30	Takayuki Koike (Osaka City University) Gluing construction of K3 surfaces and complex analysis on a neighborhood of a complex submanifold
$14:45 \sim 15:45$	Ivan Cheltsov (University of Edinburgh) Local (intersection) inequalities
$16:00 \sim 17:00$	Hirokazu Nasu (Tokai University) Obstructions to deforming curves on an Enriques-Fano 3-fold

# Oct. 25 (Fri)

8:50 ~ 9:50 Adrien Dubouloz (Université de Bourgogne)  $\mathbb{A}^1$ -cylinders over smooth affine surfaces of negative Kodaira dimension

Trains from Kinosaki Station

10:39 : Kinosaki 12 bound for Kyoto Station

11:33 : Konotori 14 bound for Shin-Osaka Station

12:32 : Kinosaki 16 bound for Kyoto Station

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::: Organizers ::: Hiraku Kawanoue (Chubu University) Takashi Kishimoto (Saitama University) Shingo Taki (Tokai University)

# Abstracts

#### Hideo Kojima

Some results on open algebraic surfaces of logarithmic Kodaira dimension zero.

We discuss open algebraic surfaces of logarithmic Kodaira dimension zero. After recalling structure theorems for open algebraic surfaces, we give a classification of open algebraic surfaces of logarithmic Kodaira dimension zero with low defects. Furthermore, by using this result, we give some results on complements of plane curves.

#### Masaru Nagaoka

Compactifications of affine homology 3-cells into quadric fibrations.

Hirzebruch raised the problem to classify all the compactifications of the affine space  $\mathbb{A}^n$  into smooth complete complex manifolds with the second Betti number  $B_2 = 1$ . A trivial example of such compactifications is the pair of the projective space and its hyperplane  $(\mathbb{P}^n, \mathbb{P}^{n-1})$ . Non-trivial examples appear in dimension three, and when the ambient spaces are projective, the complete classification of compactifications was achieved by Furushima, Nakayama, Peternell and Schneider. When  $B_2 = 2$ , non-trivial compactifications appear even in dimension two, which was studied by Mori.

In this talk, I will deal with compactifications of affine homology 3- cells, which have the same homologies as  $\mathbb{A}^3$ , into quadric fibrations such that the boundary divisors contain fibers. I will explain that all such affine homology 3-cells are isomorphic to  $\mathbb{A}^3$  and all such compactifications can be connected by explicit elementary links preserving  $\mathbb{A}^3$  to  $\mathbb{P}^3$ .

#### Shihoko Ishii

A description of divisors by arcs and its applications.

I will describe a prime divisor over a variety with a smooth center by means of an arc, more concretely a closed subset of the arc space. By this, one obtain a sufficient condition for a divisor to have a good property in the view point of lifting functions from positive characteristic to characteristic 0. If every prime divisor has the "good property", some statements for singularities in characteristic 0 are inherited to singularities in positive characteristic by modulo p reduction.

#### Joonyeong Won

Sasaki-Einstein metrics on simply connected rational homology 5-spheres.

By developing the method introduced by Kobayashi in 1960's, Boyer, Galicki and Kollár found many examples of simply connected Sasaki-Einstein 5-manifolds. For such examples they verified existence of orbifold Kähler-Einstein metrics on various log del Pezzo surfaces via links. By the recent development of method to verify existence of orbifold Kähler-Einstein metrics, we complete the classification of simply connected rational homology 5-spheres that admits Sasaki-Einstein metrics. This is a joint work with Jihun Park.

#### Takuzo Okada

Birationally superrigid Fano threefolds of codimension 4.

For a Fano variety of Picard number 1, birational (super)rigidity means a uniqueness of a Mori fiber space structure in the birational equivalence class. We consider Fano threefolds embedded in a weighted projective space. In codimensions 1, 2, 3, classification of Fano 3-folds is completed and birational (super)rigidity of them is well studied. I will review these known results in codimensions 1, 2, 3 and then I will talk about birationally superrigidity of Fano threefolds of codimension 4.

## Ken-ichi Yoshikawa

Enriques 2n-folds and analytic torsion.

In this talk, a compact connected Kähler manifold of even dimension is called simple Enriques if it is not simply connected and its universal covering is either Calabi-Yau or hyperkähler. These manifolds were introduced and studied independently by Boissière-Nieper-Weisskirchen-Sarti and Oguiso-Schröer. We introduce a holomorphic torsion invariant of simple Enriques 2n-folds and study the corresponding function on the moduli space of such manifolds. In the talk, we report its basic properties such as the strong plurisubharmonicity and the automorphy, as well as possible (conjectural) applications. If time allows, we will also report the explicit formula for the invariant as an automorphic function on the moduli space in some cases.

#### Gregory Kumar Sankaran

Locally symmetric varieties and holomorphic symplectic manifolds.

Holomorphic symplectic manifolds are generalisations of K3 surfaces to higher dimension. Their moduli spaces are related to locally symmetric varieties, in this case associated with orthogonal groups. I will describe some problems arising from the interplay between these varieties, and some recent progress.

#### Shigefumi Mori

Extremal contractions of 3-folds with fiber dimension 1.

Although the Minimal Model Program (MMP) has become a fundamental tool to study birational geometry via MMP with scaling, extremal contractions are not explicitly described even in dimension 3. This is a survey of the joint work with Dr. Y. Prokhorov on the classification of such contractions with fiber dimension 1.

#### Takayuki Koike

Gluing construction of K3 surfaces and complex analysis on a neighborhood of a complex submanifold.

We explain our recent study on the complex analytic structure of a small tubular neighborhood of a complex submanifold, which is based on T. Ueda's classification theory. We also explain how to apply them to a study on non-projective and non-Kummer K3 surfaces.

#### Ivan Cheltsov

Local (intersection) inequalities.

We will present new local inequalities that relate intersection multiplicities of curves on surfaces with basic singularities of pairs (klt/log canonical). As an application, we show how to compute delta-invariant of some log del Pezzo surfaces.

#### Hirokazu Nasu

Obstructions to deforming curves on an Enriques-Fano 3-fold.

An Enriques-Fano 3-fold is a 3-dimensional (singular) projective variety having a hyperplane section that is a smooth Enriques surface. In 1962, Mumford first gave an example of a generically non-reduced component of the Hilbert scheme of space curves. After reviewing this example and its generalizations to the Hilbert scheme of curves in smooth Fano 3-folds, I will discuss the existence of such a component of the Hilbert scheme of Enriques-Fano 3-folds.

#### Adrien Dubouloz

 $\mathbb{A}^1$ -cylinders over smooth affine surfaces of negative Kodaira dimension.

The Zariski Cancellation problem for smooth affine surfaces asks whether two such surfaces whose products with the affine line are isomorphic are isomorphic themselves. By results of Iitaka-Fujita, the answer is positive for surfaces of non-negative Kodaira dimension. By a characterization due to Miyanishi, surfaces of negative Kodaira dimension are fibered by the affine line, and by a celebrated result of Miyanishi-Sugie, the answer to the problem is positive if one of the surfaces is the affine plane. On the other hand, examples of non-isomorphic  $\mathbb{A}^1$ -fibered affine surfaces with isomorphic  $\mathbb{A}^1$ -cylinders were first constructed by Danielewski in 1989, and then by many other authors. All these counterexamples are essentially constructed by variants of the method employed by Danielewski, nowadays known as the "Danielewski fiber product trick". In this talk, I will explain that this method is actually more than a trick: re-interpreted in a suitable way, it provides a necessary and sufficient criterion for two  $\mathbb{A}^1$ -fibered surfaces over a same affine base curve C to have isomorphic relative  $\mathbb{A}^1$ -cylinders over C. The characterization can be roughly stated as follows: two such surfaces have isomorphic relative  $\mathbb{A}^1$ -cylinders over C if and only if their respectively relative (log)-canonical classes are equal when viewed as certain naturally defined  $\mathbb{Q}$ -divisors on a non-separated orbifold curve  $\check{C}$  dominating C, fully determined by the structure of the fibers of the  $\mathbb{A}^1$ -fibrations at hand. (Joint work in progress with S. Kaliman and M. Zaidenberg).

# Poster session

# **Tatsuro Kawakami** (The University of Tokyo) Bogomolov-Sommese type vanishing on globally F-regular threefolds.

標数0の滑らかな射影多様体の微分形式の層 $\Omega^i$ に含まれる直線束の飯高次元はi以下となる. これはBogomolov-Sommese vanishing と呼ばれ, Miyaoka-Yauの不等式の証明などで重要な役割を果たす. 一方,正標数ではこのBogomolov-Sommese vanishing は成立しない事が知られている.興味深いことに、多様体がWitt環にリフトしても成立しない例がある. ここで,正標数の多様体の重要なクラスとしてF分裂多様体というものがあり,この多様体上では小平の消滅定理を始めとする強力な消滅定理が成立する. さらにF分裂多様体は $W_2$ 環にリフトすることも知られている. 従って,F分裂多様体上で、Bogomolov-Sommese vanishing が成立するかという自然な疑問が生まれる. 今回の発表では、F分裂多様体の特殊なクラスである大域的F 正則多様体上で Bogomolov-Sommese vanishing について考察する.

## Yusuke Sato (The University of Tokyo)

Multi-dimensional continued fractions for Gorenstein cyclic quotient singularities.

ゴレンシュタイン巡回商特異点のクレパント特異点解消の存在性の判定法を高次元連分 数展開を用いて与える

# Yuri Shimizu (Tokyo institute of technology)

Relative  $A^1$ -homology and its applications.

*A*<sup>1</sup> ホモトピー論とは滑らかな代数多様体のホモトピー論であり、アフィン直線が単位閉 区間の代わりを果たす。本研究では、*A*<sup>1</sup> ホモトピー論において通常のホモロジーの役割を 担っている *A*<sup>1</sup> ホモロジーの相対版を考え、そのいくつかの応用と計算例を与えている。

# Takanori Nagamine (Niigata University)

A note on retracts of polynomial rings in three variables.

In Costa's paper published in 1977, he asks us whether every retract of  $k[x_1, \ldots, x_n]$  is also the polynomial ring or not, where k is a field. This question is related to the Zariski's cancellation problem as below: If Costa's question holds true for the polynomial ring in n + 1 variables, then the Zariski's cancellation problem has an affirmative answer for  $\mathbb{A}^n$ .

In this talk, we give an affirmative answer in the case where k is a field of characteristic zero and n = 3.

Yuki Tsutsui (The University of Tokyo) Radiance obstructions of tropical Kummer surfaces and their quotients.

トロピカル代数幾何とは代数幾何の組み合わせ的な類似物を研究する分野である。本ポ スター発表ではトロピカルトーラスの商としてトロピカルクンマー曲面などを構成し、周期 の類似物であるレイディアンス障害とトロピカル曲面の格子理論について考察する。

#### Shou Yoshikawa (The University of Tokyo)

Kawaguchi-Silverman conjecture on smooth rationally connected varieties admitting an int-amplified endomorphism.

We proved the Kawaguchi-Silverman conjecture for all endomorphisms on smooth rationally connected varieties admitting an int-amplified endomorphism. If X has an intamplified endomorphism, then for every endomorphism of X we consider an equivariant minimal model program (MMP). Hence by using MMP, we reduce the problem to a problem on relatively easier varieties. This is joint work with Yohsuke Matsuzawa.

**Yusuke Yoshida** (Hiroshima University) Curves whose automorphism groups are  $A_6$ .

Automorphism groups of complex projective plane curves have long been studied. Recently, a classification of automorphism groups of smooth plane curves of degree more than 4 was given. For some groups G in the classification, we studied which curves have Gas their automorphism groups. In this presentation, we state the necessary and sufficient condition on d for the existence of a smooth curve of degree d whose automorphism group is the alternating group  $A_6$ . We also state a result on irreducible curves which are invariant under  $A_6$ .

## Toshi Sugiyama (Gifu Pharmaceutical University)

The Moduli Space of Polynomial Maps and Their Fixed-Point Multipliers.

次数 d の複素 1 変数多項式写像のアファイン共役類に対して、固定点における微分係数 の値の組を対応させる写像を考える。このとき、下記 2 つの論文において、この写像のファ イバー構造を詳細に調べ上げた。特に、各ファイバーの元の個数を完全に求めた。ポスター 掲示において、その結果紹介を行う。

T. Sugiyama, The moduli space of polynomial maps and their fixed-point multipliers. Adv. Math. 322 (2017), 132–185

T. Sugiyama, The moduli space of polynomial maps and their fixed-point multipliers: II. Improvement to the algorithm and monic centered polynomials. arXiv:1802.07474, 1–18

#### Hiroto Akaike (Osaka University)

Slope inequalities for irregular cyclic covering fibrations.

代数曲線束には、スロープという相対数値不変量が定義されます.代数曲線束の幾何学的な構造と、スロープの関係については、様々な研究がされております.そこで私は、「線 織面のn次巡回被覆で,相対不正則数が正である」という幾何学的な条件を満たす代数曲線 束のスロープの下限を導出しました.この下限は,被覆次数n,一般ファイバーの種数,相対 不正則数により与えられます.

#### Tomoya Ohnishi (Kyoto University)

Arakelov geometry over a trivially valued field.

体に自明な付値を与えこれに付随するベルコビッチ空間を考えることで,任意の体上で アラケロフ幾何の理論を展開できる.本発表では基本的な理論の紹介,特にアデリック因子 の体積関数の性質についての紹介を行う.

## Keisuke Miyamoto (Osaka University)

The Sarkisov program on log surfaces.

藤野・田中により,任意標数の代数閉体上の曲面における極小モデル理論は log canonical 曲面と Q-factorial 曲面の2つのクラスで証明された.今回のポスター発表では,この2つの クラスでサルキソフ・プログラムが機能することを示す.

#### Riku Kudou (Waseda University)

Generalized Zariski cancellation problem and principal  $\mathbb{G}_a$  bundles.

Generalized Zariski cancellation problem asks whether or not  $V \times \mathbb{A}^1 \simeq W \times \mathbb{A}^1$ implies  $V \simeq W$  for varieties V and W, and counter examples for this problem have been constructed as principal  $\mathbb{G}_a$  bundles over pre-varieties. For this problem, we have the following result: if an affine variety V has a principal  $\mathbb{G}_a$  bundle structure over a pre-variety X that has some non-uniruledness properties, then for an affine variety W,  $V \times \mathbb{A}^1 \simeq W \times \mathbb{A}^1$  if and only if W has a principal  $\mathbb{G}_a$  bundle structure over X.

#### Rikito Ohta (Osaka University)

On Seshadri constants of non-simple abelian varieties.

アーベル多様体上の豊富直線束の Seshadri 定数が直線束の次数と比べて十分に小さい場合にそれは元の多様体のある部分アーベル多様体の Seshadri 定数と一致していることが分かった。また、それとは独立に一定の条件を満たす余次元1の部分アーベル多様体を持つ場合にも元の多様体の Seshadri 定数がその部分多様体の Seshadri 定数と一致する事が示せた。 今考えている応用も含めてこれらのことを発表する。

# Niklas Lemcke (Waseda University)

Duality for Witt sheaves.

I show a duality property in the setting of [Tanaka17], in the attempt to generalize Tanaka's vanishing theorem to nef and big line bundles.

[Tanaka17] - Tanaka Hiromu, Vanishing theorems of Kodaira type for Witt canonical sheaves, 2017 (https://arxiv.org/abs/1707.04036)

## Yuta Kambe (Saitama University)

Computable Bialynicki-Birula decomposition of the Hilbert scheme.

ヒルベルトスキームを適当なグラスマン多様体に埋め込んで考えると、その有理点集合 はある条件を満たすイデアルの集合と考えることができる。したがって、項順序を一つ多項 式環に固定したとき、ヒルベルトスキームの有理点集合は各点に対応するイデアルのイニ シャルイデアルが何であるかによって分割される。その分割は計算機によって計算可能であ り、またいわゆる Bialynicki-Birula 分割であることを紹介する。その応用として、なめらか なヒルベルトスキームのホモロジー群公式や、単項式イデアルで定まる特異点の計算方法を 紹介する。

**Takahiro Nagaoka** (Kyoto university) The universal cover of hypertoric varieties and Bogomolov's decomposition.

Hypertoric varieties give very tractable examples of holomorphic symplectic varieties like toric varieties. Recently, deformations and resolutions of holomorphic symplectic varieties have been extensively studied. In this poster, we will study the (singular) universal cover of hypertoric varieties and the existence of an analogue of Bogomolov's decomposition of this universal cover.

**Taiki Takatsu** (Tokyo Institute of Technology) On the geometry of singular K3 surfaces with discriminant 3, 4 and 7.

**Yuto Yamamoto** (IBS Center for Geometry and Physics) *Periods of tropical Calabi-Yau hypersurfaces.* 

トロピカル Calabi-Yau 超曲面を収縮してできる、特異性を持つ整アフィン構造付き S<sup>n</sup> の radiance obstruction を計算して、それが複素 Calabi–Yau 超曲面の周期写像の leading term を与えることを示します。